03E_At the Crack Tip - CTOD

Summary:

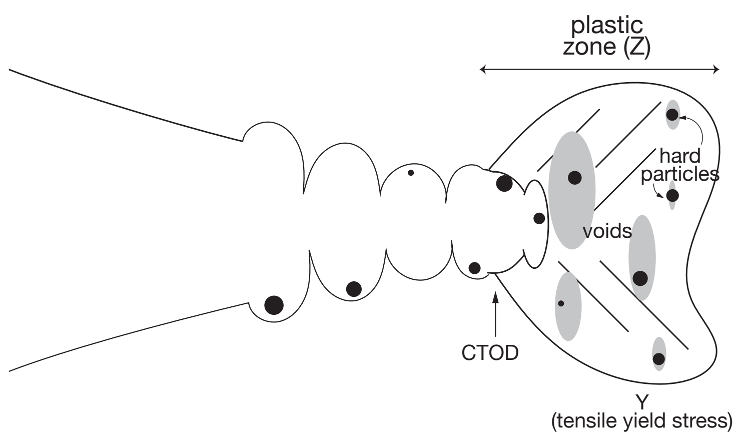
The fracture criterion at the crack tip can be prescribed as a critical value of either the local tensile stress, or the CTOD (crack tip opening displacement).

While the stress criterion is appropriate for highly brittle materials where the crack moves when the force on the bonds just ahead of the crack tip exceeds the break point, the stress criterion is not appropriate when there is plastic deformation at the crack tip, since simply increasing the yield stress does not cause fracture: the material at the crack tip must be torn apart as in in the neck region of a uniaxial test.

Therefore, in cases where there is plastic yielding (or stretching of fibrils in a polymer), or some other kind of damage accumulation at the crack tip which depends on the local tensile displacement, a critical value of the crack tip opening displacement (CTOD) is the appropriate fracture criterion.

The case of small scale yielding in a "hard" metal is shown below, but a similar sketch can be developed for polymers, where fibrils are stretched to fracture, or in ceramics where there is limited damage that extends farther into the material from the crack tip. The analysis described below applies to all these scenarios*, although we shall consider the plastic yielding in metals as the illustrative example, as sketched below.

*As a general statement, " the crack must open to a certain extent to accommodate the damage at the crack tip so that the crack can advance".



The analysis given below invokes the following parameters:

σ_Y the ultimate yield stress (the maximum engineering stress in a uniaxial tensile test)

CTOD fracture occurs when CTOD reaches a critical value

Z the length scale of the damage zone

E the elastic modulus of the metal since fracture always depends on the stored elastic energy.

Starting equation $2\gamma_F = \dfrac{K_{IC}^2}{E}$ (A)

Our objective is to get and expression for K_{IC}

The Approach

To analyze for $2\gamma_F$, and then get an expression for K_{IC}

Consider the work of fracture in terms of units.

We know that the work of fracture would be work done per unit area of crack extension.

Let us say the thickness of the crack normal to the paper is = w

Then the effective area of the plastic zone = Zw

Work done = force on the plastic zone * displacement

$$= \sigma_Y Zw \; * \; CTOD \qquad (1)$$

The work done to advance the crack by one unit area = $2\gamma_F * Zw$ (2)

Equating (1) and (2)

$$2\gamma_F = \sigma_Y (CTOD) \qquad (3)$$

Equating (A) and (3)

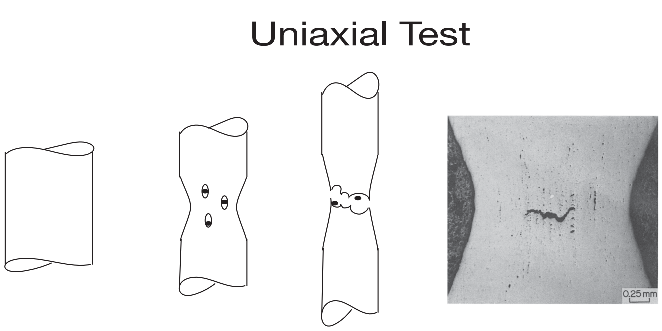
$$K_{IC}^2 = \sigma_Y E(CTOD) \qquad (4)$$

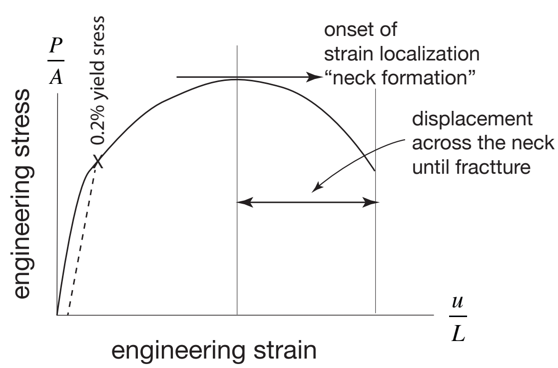
To some extent the yield stress and the displacement to fracture can be estimated in a tensile test, but practically speaking the CTOD is measured experimentally but watching the crack tip, which is then compared with prediction from Eq. (4).

The prediction from Eq. (4) is from the measurement of K_{IC}, the yield stress (in a simple uniaxial tensile test) and the Elastic modulus (a handbook value).

The prediction from Eq. (4) of the CTOD can be compared from optical microscope observations of the crack tip.

Consider a Simple Uniaxial Tensile Test





Fracture in a simple uniaxial test consists of three stages: the localization of strain within a neck region, the growth of damage, followed by rupture. The onset of localization is marked by a maximum in the engineering stress in a tensile stress. This is effectively the yield stress within the damage zone.

The applied uniaxial displacement once the neck has formed would correspond approximately to the CTOD for crack propagation - but this is difficult to pin down since some of the applied displacement is related to the regions away from the neck along the gage length of the tensile specimen.

Therefore it is better to measure K_{IC} and then calculate CTOD from Eq. (4) and compare with experiment.

Experimental Measurement of CTOD from Eq. (4)

Please refer to two papers attached to this write-up and posted on the website. They are:

03E1: Hahn and Rosenfield (pictures of crack tip opening - Si+C steel)
03E2: Putatunda (Yield strength and fracture toughness - Si+C steel)

Data from 03E2 for yield strength and fracture toughness are given below:

Table 2
Mechanical properties of HCHS steel

Austempering temperature (°C)	Yield strength (MPa)	Tensile strength (MPa)	Elongation (%)
260	1800 ± 55	2100 ± 61	1.8 ± 0.5
288	1750 ± 60	2090 ± 63	2.1 ± 1.2
302	1645 ± 49	2010 ± 54	3.2 ± 1.5
316	1550 ± 51	1930 ± 46	4.2 ± 3.5
357	1480 ± 36	1670 ± 51	5.1 ± 2.5
371	1310 ± 42	1490 ± 38	6.2 ± 1.8
385	1060 ± 46	1310 ± 26	7.0 ± 2.6
399	988 ± 31	1088 ± 49	2.3 ± 1.3

The tensile strength is the peak value in the stress-strain curve.

Let us consider the case of the specimen austempered at 300 °C with a

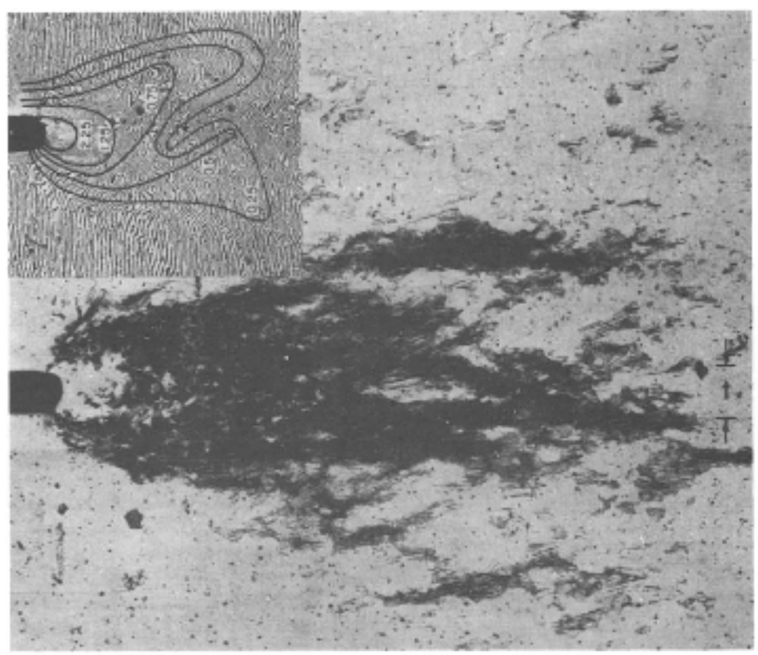
•tensile strength of ~2000 MPa and
•fracture touchness of ~60 MPa $m^{i/2}$.
Young's modulus of high strength steel is 190 GPa (actually it ranged from 180-200 GPa)

Substituting these values into Eq. (4):

$$CTOD = \frac{K_{IC}^2}{\sigma_Y E}$$

High Strength Steel

K_IC	60	MPa m^1/2
E	190	GPa
	190000	MPa m^1/2
yield stre	2000	MPa
CTOD	9.47368E-06	m
	9.473684211	um



That is the CTOD is approximately 10 μm.

Now let us compare with the pictures in 03E1
The marker "t" in the micrograph is equal to 0.017", that is 17*25 μm

Therefore the predicted CTOD would be about (1/35) of the scale marked on the right hand side in the figure. It is difficult to say where the CTOD is. If it is the small narrow streak aligned at about 36 ° to the plane of the crack (right at the crack tip) then it would seem that there is reasonable agreement between theory and experiment.